Progress in Mathematics

The Breadth of Symplectic and Poisson Geometry

Festschrift in Honor of Alan Weinstein

Jerrold E. Marsden
Tudor S. Ratiu
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Margo, Alan, and Asha in Paris at the lovely Fontaine des Quatre Parties du Monde.

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Preface

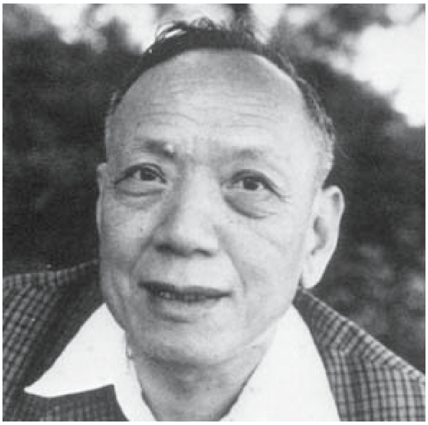
Alan Weinstein is one of the top mathematicians in the world working in the area of symplectic and differential geometry. His research on symplectic reduction, Lagrangian submanifolds, groupoids, applications to mechanics, and related areas has had a profound influence on the field. This area of research remains active and vibrant today and this volume is intended to be a reflection of that vigor. In addition to reflecting the vitality of the field, this is a celebratory volume to honor Alan's 60th birthday. His birthday was also celebrated in August, 2003 with a wonderful week-long conference held at the ESI: the Erwin Schrödinger International Institute for Mathematical Physics in Vienna.

Alan was born in New York in 1943. He was an undergraduate at MIT and a graduate student at UC Berkeley, where he was awarded his Ph.D. in 1967 under the direction of S. S. Chern. After spending postdoctoral years at IHES near Paris, MIT, and the University of Bonn, he joined the faculty at UC Berkeley in 1969, becoming a full Professor in 1976.

Alan has received many honors, including an Alfred P. Sloan Foundation Fellowship, a Miller Professorship (twice), a Guggenheim Fellowship, election to the American Academy of Arts and Sciences in 1992, and an honorary degree at the University of Utrecht in 2003.

At the ESI conference, S. S. Chern, Alan's advisor, sent the following words to celebrate the occasion:

> "I am glad about this celebration and I think Alan richly deserves it. Alan came to me in the early sixties as a graduate student at the University of California at Berkeley. At that time, a prevailing problem in our geometry group, and the geometry community at large, was whether on a Riemannian manifold the cut locus and the conjugate locus of a point can be disjoint. Alan immediately showed that this was possible. The result became part of his Ph.D. thesis, which was published in the *Annals of Mathematics*. He received his Ph.D. degree in a short period of two years. I introduced him to IHES and the French mathematical community. He stays close with them and with the mathematical ideas of Charles Ehresmann. He is original and



often came up with ingenious ideas. An example is his contribution to the solution of the Blaschke conjecture. I am very proud to count him as one of my students and I hope he will remain interested in mathematics up to my age, which is now 91."

Alan's technical contributions are wide ranging and deep. As many of his early papers in his publication list illustrate, he started off in his thesis and the years immediately following in pure differential geometry, a topic he has come back to from time to time throughout his career.

Already starting with his postdoc years and his early career at Berkeley, he became interested in symplectic geometry and mechanics. In this area he rapidly established himself as one of the world's authorities, producing important and deep results ranging from reduction theory to Lagrangian and Poisson manifolds to studies of periodic orbits in Hamiltonian systems. He also did important work in fluid mechanics and plasma physics and through this work, he established warm relations with the Berkeley physicists Allan Kaufman and Robert Littlejohn.

Alan's important work on periodic orbits in Hamiltonian systems led him eventually to formulate the "Weinstein conjecture," namely that for a given Hamiltonian flow on a symplectic manifold, there must be at least one closed orbit on a regular compact contact type level set of the Hamiltonian. Along with Arnold's conjecture, the Weinstein conjecture has been one of the driving forces in symplectic topology over the last two decades.

Alan kept up his interest in symplectic reduction theory throughout his later work. For instance, he laid some important foundation stones in the theory of semidirect product reduction as well as in singular reduction through his work on Satake's V-manifolds, along with finding important links with singular structures in moduli spaces.

Intertwined with his work on symplectic geometry and mechanics, he did extensive work on geometric PDE, eigenvalues, the Schrödinger operator and geometric quantization. Alan took the point of view of microlocal analysis and phase space structures in his work in this area, emphasizing the links with quantum mechanics.

His work on the limit distribution of eigenvalue clusters in terms of the geodesic Radon transform of the potential inspired a large number of related articles. He showed that the geodesic flow of a Zoll surface was symplectically equivalent to that of a round sphere, and hence that its Laplacian could be conjugated globally to the round Laplacian plus a pseudodifferential potential. This work inspired many other results on conjugacies.

One of Alan's fundamental contributions to Poisson geometry was the introduction of symplectic groupoids in 1987, which marks the official beginning of his "oids" period. In these works, he makes sweeping generalizations about a wide variety of constructions in symplectic geometry, including (with Courant) the important notion of Dirac structures. During this period of generalizations he constantly returned to specific topics in symplectic and Poisson geometry, such as geometric phases and Poisson Lie groups, in addition to making other key links. For instance, symplectic groupoids are used to link Poisson geometry to noncommutative geometry, and groupoids are also intimately related to many other areas, including symmetries and reduction, dual pairs, quantization and the theory of sigma models. One of the central ideas is that the usual theory of Hamiltonian actions, momentum maps, and symplectic reduction makes sense in the more general context of actions of symplectic groupoids; in this setting, momentum maps are Poisson maps taking values in general Poisson manifolds, rather than just Lie–Poisson manifolds (that is, duals of Lie algebras). Alan has raised the question of whether this framework can be further extended to include new notions of momentum maps such as quasi-Poisson manifolds with group-valued momentum maps as well as optimal momentum maps.

Alan is well known not only for his brilliant papers and conjectures, but also for his general philosophy, such as the symplectic creed: *Everything is a Lagrangian submanifold*. Those of us who know him well also appreciate his very special insight. For example, in the middle of a discussion (for instance, as we both had in our joint works on semidirect product reduction as well as stability theory) he will say something like *what you are really doing is...* and then give us some usually very special insight that invariably substantially improves the whole project.

Alan also has a very interesting and charming sense of humor that even makes its way into his papers from time to time. For instance, Alan had great fun in his papers with the "East Coast–West Coast" discussions of whether one should use the term *momentum map* or *moment map*. He also gave us a good laugh with the term *symplectic bones* as it relates to the French translation of *Poisson* as *Fish*.

Alan is a great educator. His lectures, even on Calculus, are always a treat and are very inspiring for their special insight, their wit and lively presentation. His enthusiasm for mathematics is infectious. One story that comes to mind on the education front is this: during the days when he was exceptionally keen about groupoids, he was preparing a lecture for undergraduates on the subject. Some of us convinced him to present it as a colloquium lecture for faculty, keeping in mind the old advice "no colloquium talk can be too simple." It was, in fact, not only a beautiful colloquium talk, but was perfectly pitched for the faculty, and it became a popular article in the *Notices of the American Mathematical Society*. Part of being a good educator is being cognizant of history. Alan excels in this area. For instance, his research into the history of Lie is what led directly to the introduction of the term “Lie–Poisson” bracket.

The papers in this volume were selected by invitation and all of them underwent a rigorous refereeing process. While this process took some time, it resulted in high quality papers. We thank all of the referees for their diligent and helpful work. The authors of this volume represent some of the best workers in the subject and their contributions span a wide range of the topics covered by symplectic and Poisson geometry and mechanics, broadly interpreted.

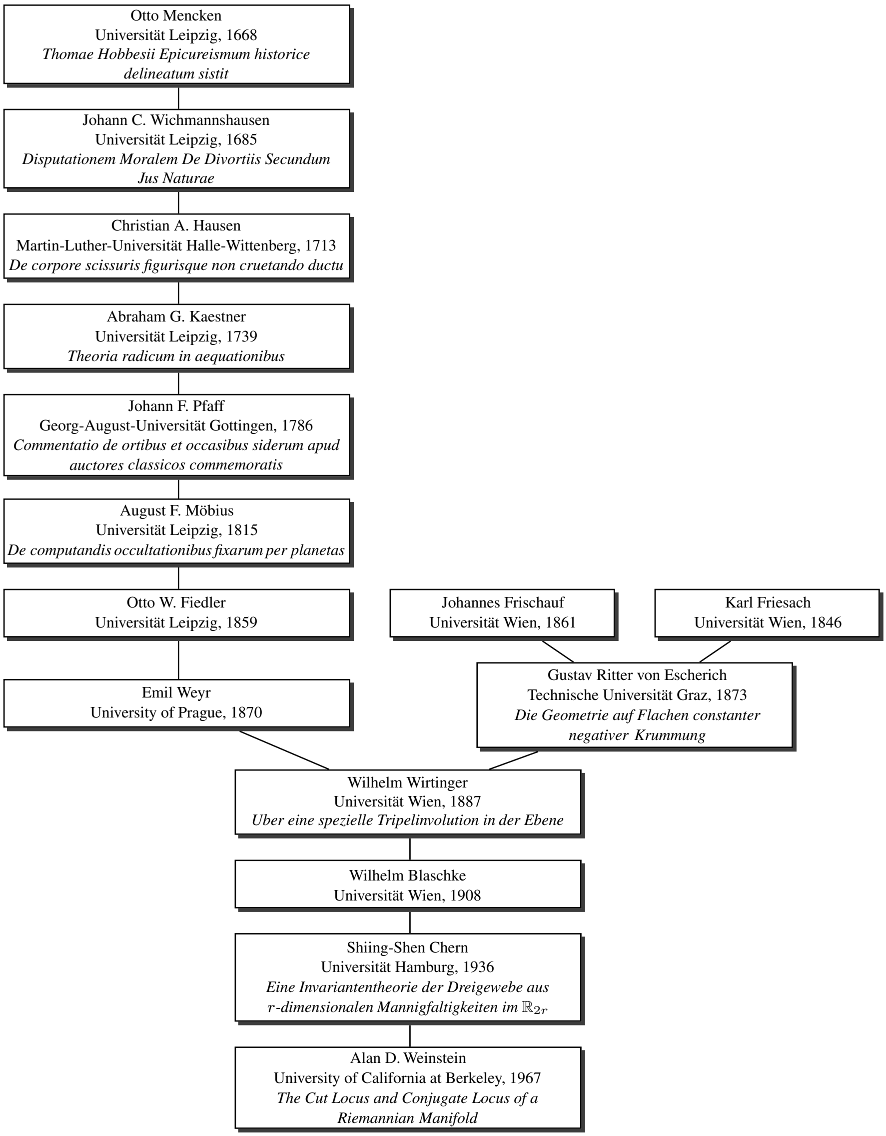
The intended audience for the book includes active researchers in the general area of symplectic geometry and mechanics, as well as aspiring graduate students who wish to learn where the subject is headed and what some of the current research topics are.

Alan and Margo have a special relationship to Paris. They have spent many happy times there, and we wish them all the best and many more happy visits in the years to come.

We wish to thank Ann Kostant for her expert editorial guidance throughout the production of this volume. Of course, we also thank all the authors for their contributions as well as their helpful guidance and advice. The referees are also thanked for their valuable comments and suggestions.

Jerry Marsden and Tudor Ratiu
September, 2004

Academic genealogy of Alan Weinstein



About Alan Weinstein

Alan David Weinstein

Ph.D.: University of California at Berkeley, 1967
Dissertation: *The Cut Locus and Conjugate Locus of a Riemannian Manifold*
Advisor: Shiing-Shen Chern

Students of Alan Weinstein

1. Jair Koiller, *Studies on the Spring-Pendulum*, 1975
2. Otto Ruiz, *Existence of Brake-Orbits in Finsler Mechanical Systems*, 1975
3. Yilmaz Akyildiz, *Dynamical Symmetries of the Kepler Problem*, 1976
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5. John Jacob, *Geodesic Symmetries of Homogeneous Kahler Manifolds*, 1977
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7. Enrique Planchart, *Analogies in Symplectic Geometry of Some Results of Cartan in Representation Theory*, 1982
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10. Theodore Courant, *Dirac Manifolds*, 1987
11. Yong-Geun Oh, *Nonlinear Schrödinger Equations with Potentials: Evolution, Existence, and Stability of Semi-Classical Bound States*, 1988
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14. Jiang-Hua Lu, *Multiplicative and Affine Poisson Structures on Lie Groups*, 1990
15. Ping Xu, *Morita Equivalence of Poisson Manifolds*, 1990