FOREWORD

The introduction of differentials by Leibniz and Newton might be considered as the first appearance of infinitesimals in the mathematics of modern times, famous also because these objects were soon derided by Berkeley as ghosts of a highly unsure existence. Infinitesimals, i.e., non-vanishing positive 'numbers' smaller than any given fraction, had still been very useful for Euler and Cauchy; they became ruled out only later on, in the wake of the putatively rigorous tendencies which took over mathematical analysis during the 19th century. Since, the best known renewal of that venerable concept has been carried out in the preceding four decades by Abraham Robinson and others, who designed this non-standard analysis to deal with infinitely small and infinitely large entities in a truly rigorous manner. Because of making a rather unrestricted use of classical logic and set theory and, in particular, of the axiom of choice, Robinson's theory in its full-fledged form has widely been suspected to be nonconstructive from the outset. In addition, the nonstandard idea of discretising the continuum seems to be even less compatible with the intuitionistic concept of a continuum in the true sense of the word, than with the classical atomistic notion.

The distance between constructive and nonstandard mathematics, however, is actually much smaller than it appears to be. Indications for this are that nonstandard practice often looks rather constructive, and that very small numbers unknown to vanish are indispensable to distinguish constructive mathematics from its traditional counterpart. At least from any naive point of view, it is therefore no wonder that constructive mathematics eventually proved its capability to tackle also relatively abstract objects such as infinitesimals. This progress cannot be thought of without the revival of constructive thinking since the 1960es, initiated by the work of Errett Bishop, Per Martin-Löf, and others as well as by the development of digital computers, which has eventually lead to today's pragmatic way in which constructive mathematics sees itself. Some far-reaching approaches to constructive nonstandard mathematics have indeed been undertaken quite recently, whence time was ripe for the first meeting dedicated simultaneously to constructive and nonstandard mathematics—and, of course, to the reunion of these seeming antipodes.

Consisting of peer-reviewed articles written on the occasion of such an event, this volume offers views of the continuum from various standpoints. Including historical and philosophical issues, the topics of the contributions range from the foundations, the practice, and the applications of constructive and nonstandard mathematics, to the interplay of these areas and the development of a unified theory.

Further Talks.   The following talks were given at the conference but will be published elsewhere.

PETR ANDREYEV (Nizhnii Novgorod State University, Russia), *Definable standardness predicates in Internal Set Theory*

DOUGLAS BRIDGES (University of Canterbury, Christchurch, New Zealand), *Constructive investigations of functions of bounded variations*

NIGEL J. CUTLAND (University of Hull, England), *Constructive aspects of nonstandard methods in fluid mechanics*

DIRK VAN DALEN (Rijksuniversiteit Utrecht, The Netherlands), *Indecomposable subsets of the continuum*

JENS ERIK FENSTAD (University of Oslo, Norway), *Computability theory over the nonstandard reals*

JAMES HENLE (Smith College, Northhampton, Massachusetts), *Non-nonstandard analysis: category, measure, and integration*

CHRIS IMPENS (University of Gent, Belgium), *Some thoughts on non-standard geometry*

HAJIME ISHIHARA (Japan Advanced Institute of Science and Technology), *A note on the Gödel-Gentzen translation*

H. JEROME KEISLER (University of Wisconsin, Madison), *Nonstandard methods in o-minimal structures*

M. ALI KHAN (John Hopkins University, Baltimore, Maryland), *Modelling 'neglibility' in mathematical economics: an application of Loeb spaces*

P. EKKEHARD KOPP (University of Hull, England), *Hyperfinite discretisations and convergence in option pricing models*

XIAOAI LIN (National University of Singapore), *On the almost independence of correspondences on Loeb space*

WILHELMUS A. J. LUXEMBURG (California Institute of Technology, Pasadena), *A Schauder type theorem for internal linear operators*

PER MARTIN-LÖF (University of Stockholm, Sweden), *Nonstandard type theory*

JOAN R. MOSCHOVAKIS (Occidental College, Los Angeles; University of Athens, Greece), *The intuitionistic continuum as an extension of the classical one*

JUHA OIKKONEN (University of Helsinki, Finland), *Some geometric ideas related to Brownian motion*

HERVÉ PERDRY (Université de Franche-Comte, Besançon, France), *Computing in the constructive henselisation of a valued field*

HANS PLOSS (Universität Wien, Austria), *On the rearrangement of series*

MICHAEL REEKEN (Universität Wuppertal, Germany), *Discretising the continuum*

HERMANN RENDER (Universität Duisburg, Germany), *Borel measure extensions defined on sub-σ-algebras*

GIOVANNI SAMBIN (Università di Padova, Italy), *Real numbers in formal topology*

PETER SCHUSTER (Universität München, Germany), *Elementary choiceless constructive analysis*

BAS SPITTERS (Katolieke Universiteit Nijmegen, The Netherlands), *A constructive converse of the mean value theorem*

YENENG SUN (National University of Singapore), *Asymptotic, hyperfinite, and continuum models*

WIM VELDMAN (Katolieke Universiteit Nijmegen, The Netherlands), *On some sets that are not positively Borel*

HANS VERNAEVE (University of Gent, Belgium), *Reducing distributions to hyperreal functions*

MANFRED WOLFF (Universität Tübingen, Germany), *On the approximation of operators and their spectra*

Further Participants

STEFANO BARATELLA (Università di Trento, Italy)

JOSEF BERGER (Universität München, Germany)

JENS BLANCK (University of Wales, Swansea)

JONAS ELIASSON (University of Uppsala, Sweden)

FERRUCCIO GUIDI (Università di Padova, Italy)

GÖRAN HAMRIN (University of Uppsala, Sweden)

MARTIN KÜBLER (Universität München, Germany)

JUHA RUOKOLAINEN (University of Helsinki, Finland)

PETER SCHUSTER (Ludwig-Maximilians-Universität München)

ULRICH BERGER (University of Wales Swansea)

HORST OSSWALD (Ludwig-Maximilians-Universität München)

Munich and Swansea.