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Pablo Pedregal

Optimal Design through the Sub-Relaxation Method

Understanding the Basic Principles

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To my beloved sister Conchi, in memoriam

Preface

This book aims to introduce an alternative analytical method for the solution of optimal design problems in continuous media. As such, it is not meant to serve as an introductory text on optimal design. In fact, a certain degree of familiarity with more classical approaches, especially the homogenization method, is required in order to appreciate fully the comments and results. It is also assumed that the reader will have had some exposure to the significance and relevance of these problems in Engineering, as well as to the various numerical procedures developed to simulate optimal designs in practical problems. The material and the treatment are intended to be self-contained in such a way that, in addition to covering the aforementioned aspects, the book will serve as a sound basis for a masters or other postgraduate courses in the subject.

Application to real problems in Engineering would almost demand a separate book. On the one hand, many specific situations may have an interesting mechanical background (e.g., compliant mechanisms or vibrating structures), an electric/electronic flavor (e.g., optimal design with piezoelectric materials), or relevance in other fields. On the other hand, there are many delicate issues associated with computational aspects which are well beyond the scope of this work and would demand a separate contribution written by somebody with extensive expertise in those topics. We simply illustrate analytical results with some simple, academic examples and provide well-known references to cover all relevant aspects of optimal design.

The book also aims to persuade young researchers, on both the analytical and the computational side, to further pursue the development of the sub-relaxation method. I firmly believe that there is still much room for improvement. Although some new directions may be very hard to examine (e.g., the analysis for the elasticity setting and the implementation of point-wise stress constraints, to name just two important ones), others may lie within reach. In particular, applying the sub-relaxation method, appropriately adapted for numerical simulations, to realistic problems and situations may result in quite interesting approximation techniques.

Some further training in Analysis is assumed, including basic Measure Theory, Sobolev spaces, basic theory of weak solutions for equations and systems of equilibrium, weak convergence, etc. Moreover, it is desirable that the reader has some previous experience with the basic techniques of the calculus of variations, the role of convexity in weak lower semicontinuity, and how the failure of this fundamental structural property may result in special oscillatory behavior. Again, some simple discussions and examples may serve to fill this gap, and so provide the reader with a basic, well-founded intuition on these important issues.

The book is intended for masters or graduate students in Analysis, Applied Math, or Mechanics, as well as for more senior researchers who are new to the subject. At any rate, readers are expected to have sufficient analytical maturity to understand issues not fully covered here in order to appreciate the ideas and techniques that are the basis for the sub-relaxation approach to optimal design.

I would like to express my sincere gratitude to an anonymous reviewer whose positive criticism led to various significant improvements in the presentation of this text. Several colleagues from the editorial board of the Springer SEMA-SIMAI Series also helped a lot in making this project a reality. Particular thanks go to L. Formaggia and C. Pares for their specific interest in this book. F. Bonadei from Springer played an important role, too, in leading this project to final completion.

Ciudad Real, Spain Pablo Pedregal
May 2016

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Chapter 1
Motivation and Framework

1.1 The Model Problem

It is not difficult to motivate, from a practical point of view, the kind of situations we would like to deal with and analyze. We have selected a typical example in heat conduction, but many other examples are as valid as this one. Suppose we have two very different materials at our disposal: the first, with conductivity $\alpha_1 = 1$, is a good and expensive conductor; the other is a cheap material, almost an insulator with conductivity coefficient $\alpha_0 = 0.001$. These two materials are to be used to fill up a given design domain Q, which we assume to be a unit square for simplicity (Fig. 1.1), in given proportions t_1, t_0, with $t_1 + t_0 = 1$. Typically, $t_1 < t_0$ given that the first material is much more expensive than the second. We will take, for definiteness, $t_1 = 0.4$, $t_0 = 0.6$. The thermal device is isolated all over ∂Q, except for a small sink Γ_0 at the middle of the left side where we normalize temperature to vanish, and there is a uniform source of heat all over Q of size unity. The mixture of the two materials is to be decided so that the dissipated energy is as small as possible.

If we designate $u(x, y)$ as temperature, and use a characteristic function χ to indicate where to place the good conductor in Q, then we would like to find the optimal such distribution $\overline{\chi}$ minimizing the cost functional

$$\int_Q u(x, y)\, dx\, dy$$

that measures dissipated energy, among all those mixtures χ complying with

$$-\text{div}[(\alpha_1 \chi(x, y) + \alpha_0(1 - \chi(x, y)))\nabla u(x, y)] = 1 \text{ in } Q,$$

$$u = 0 \text{ on } \Gamma_0, \quad (\alpha_1 \chi(x, y) + \alpha_0(1 - \chi(x, y)))\nabla u(x, y) \cdot \mathbf{n} = 0 \text{ on } \partial Q \setminus \Gamma_0,$$

$$\int_Q \chi(x, y)\, dx\, dy = 0.4.$$

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