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Thermo-Dynamics of Plates and Shells

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Preface

The present monograph is devoted to nonlinear dynamics of thin plates and shells with termosensitive excitation. Since the investigated mathematical models are of different sizes (two- and three-dimensional differential equation) and different types (differential equations of hyperbolic and parabolic types with respect to spatial coordinates), there is no hope to solve them analytically. On the other hand, the proposed mathematical models and the proposed methods of their solutions allow to achieve more accurate approximation to the real processes exhibited by dynamics of shell (plate) - type structures with thermosensitive excitation. Furthermore, in this monograph an emphasis is put into a rigorous mathematical treatment of the obtained differential equations, since it helps efficiently in further developing of various suitable numerical algorithms to solve the stated problems.

It is well known that designing and constructing high technology electronic devices, industrial facilities, flying objects, embedded into a temperature field is of particular importance. Engineers working in various industrial branches, and particularly in civil, electronic and electrotechnic engineering are focused on a study of stress-strain states of plates and shells with various (sometimes hybrid types) damping along their contour, with both mechanical and temperature excitations, with a simultaneous account of heat sources influence and various temperature conditions. Very often heat processes decide on stability and durability of the mentioned objects. Since numerous empirical measurement of heat processes are rather expensive, therefore the advanced precise and economical numerical approaches are highly required.

A brief monograph description follows. Chapter 1 of this monograph is devoted to a study of three-dimensional problems of theory of plates in a temperature field. First, a brief historical outline as well as a state-of-art of the mentioned problems is described in introductional section. In Section 1.2, the system of differential equations governing a broad class of problems in the coupled dynamic theory of thermoelasticity in three-dimensional formulation is derived. A difference variational approximation is given and the difference scheme error is derived. Also stability of an explicit difference scheme is rigorously studied.

Section 1.3 includes a comparison of solving systems governed by either hyperbolic or elliptic equations through various iterative methods.

In section 1.4 numerous results of solutions of broad class of elasticity and thermoelasticity problems including coupling of temperature and deformations, are illustrated and discussed.

In Chapter 2, after a brief historical research review, the variational equations for shallow anisotropic shells embedded into a temperature are derived. Coupling conditions and stress-strain state of shallow shells are formulated. In section 2.2 universality and efficiency of finite difference method devoted to boundary value problems for elliptic equations if outlined. Difference schemes for series of multidimensional stationary heat transfer equations are proposed in both sections 2.2 and 2.3. In the last section 2.4, influence of heat sources on a shell stress-strain and its stability is studied.

Chapter 3 is devoted to analysis of dynamical behaviour and stability of closed cylindrical shells subject to continuous thermal load. A brief historical background is followed by variational formulation of the coupled dynamical problem of thermoelasticity. Hybrid-type variational equations of thin conical composite orthotropic thermosensitive shells are derived, and a problem of their solution is rigorously discussed. Furthermore, a solution to the biharmonic equation in relation to forcing function, as well as reliability of the obtained results, are addressed. Dynamical stability loss and non-uniform thermal loading are also studied.

Dynamical behaviour and stability of rectangular shells is addressed in Chapter 4. In section 4.1, the computational algorithm to analyse differential equations with the associated boundary conditions is derived. The associated finite difference equations are given, and reliability of the results are verified. Stationary state method to analyse statical and dynamical problems is illustrated in section 4.1.4. Various vibrational phenomena and stability loss are studied. Stability of thin shallow shells with both transversal and heat loads are examined in section 4.2. Section 4.3 is devoted to stability of thin conical shells subject to both longitudinal load and heat flow. Finally, dynamical stability of flexurable conical shells with convection is studied in section 4.4.

In Chapter 5 dynamics and stability of flexurable sectorial shells with thermal loads are addressed. First, theory of flexurable sectorial shells is introduced. The fundamental relations are assumed, differential equations are derived and initial conditions are given. After introduction of a thermal field the numerical "set-up" technique is illustrated and discussed, and numerical results reliability is outlined. Then various examples of stability of sectorial shells with finite deflections are studied. In addition, chaotic dynamics of sectorial shells and its control is addressed.

Chapter 6 is devoted to a study of coupled problems of thin shallow shells in temperature field within the Kirchhoff-Love kinematic model. Fundamental assumptions and relations are introduced, and the differential equations are derived. The finite difference model of a solution to three dimensional heat conductivity equation is formulated. Numerical algorithm to solve the obtained equations is proposed, and then numerous examples of investigation of stability loss of shallow rectangular shells follow. Additional original method to solve a coupled thermoelastic problem is also proposed.

In chapter 7 a novel optimal and exact method of solving large systems of linear algebraic equations. In the approach under consideration the solution of a system of algebraic linear equations is found as a point of intersection of hyperplanes, which needs a minimal amount of computer operating storage. The proposed method makes it possible to benefit from the essential advantages of both the direct method (universality, finitness of a computational process, exactness) and the iterational one (minimal amount of operational storage). Two examples are given. In the first example, the boundary value problem for a three-dimensional stationary heat transfer equation in a parallelepiped in $\mathbf{R}^3$ is considered, where boundary value problems of the 1st, 2nd or 3rd order, or their combinations are taken into account. The governing differential equations are reduced to algebraic ones with the help of the finite element and the boundary element methods for different meshes applied. The obtained results are compared with known analytical solutions. The second example concerns computation of a non-homogeneous shallow physically and geometrically non-linear shell subject to transversal uniformly distributed load. The partial differential equations are reduced to a system of non-linear algebraic equations with the error of $O(h_{x_1}^2 + h_{x_2}^2)$. The linearization process is realized through either Newton method or differentiation with respect to a parameter. In consequence, the relations of the boundary condition variations along the shell side and the conditions for the solution matching are reported.

In the last Chapter 8, some rigorous mathematical treatments of a coupled thermomechanical problems are addressed. First, the sufficient conditions of existence, uniqueness and continuity dependence on initial data of the Cauchy problem solutions for differential-operational equation of hybrid type (a part of the equation is of hyperbolic type, and another part is of parabolic type) are given. It is shown that if the operational coefficients are suitably chosen, the investigated equation can model a differential equations governing vibrations of a plate, i.e. the modified Germain-Lagrange equation of thermal conductivity (a parabolic equation).

Second, a coupled thermo-mechanical of non-homogeneous shells with variable thickness and variable Young modulus (a so-called Timoshenko type model) is studied. The investigated problem is reduced to uniformly correct problem in the first form of a first order difference equation.

Third, boundary conditions for a non-homogeneous first order operator – differential equation possessing a unique solution are derived. Two important theorems are formulated.

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1 Three–Dimensional Problems of Theory of Plates in Temperature Field

In section 1.1 historical outline putting emphasis on not solved problems in three-dimensional formulation of plates thermoelastic theory is given.

Section 1.2 presents a system of differential equations describing a broad class of problems of the coupled dynamic theory of thermoelasticity in a complete, three-dimensional formulation including material's non-homogeneity. The investigated system of equations has been supplemented with an equation at singular points of the examined space (a cubicoid), such as ribs, corners and simple points where various boundary conditions meet. A difference approximation of the initial differential system has been formulated with the use of the variational-difference method (the method of integral identity). The margin of the difference scheme error has been estimated. A theorem concerning stability of an explicit difference scheme has been proven and the condition of stability that guarantees weak convergence of the difference scheme's solution towards the solution of a differential system has been obtained.

Section 1.3 contains a comparison of solving systems of hyperbolic equations (using an explicit difference scheme based on applying Runge-Kutta's method with automatic choice of an integration step and Runge-Kutta's method with a constant step). Additionally, the section presents a comparison of applied iterative methods of solving systems of elliptic equations (Seidel's method, the upper relaxation, the explicit and implicit methods of variable directions, and the explicit method of variable directions with the so-called Chebyshev's acceleration). Several model problems have been used to draw the comparisons and the most economical methods have been applied as far as accuracy of solutions and computation time are concerned. Algorithms of the described methods have been formulated and a package of programs for solving problems of statics, quasistatics and elasticity and thermoelasticity dynamics has been created. An optimum choice of a spatial mesh step and an integration step within a time interval has been made and legitimacy of the theoretically obtained (in the first section) stability condition has been numerically confirmed. Feasibility of the obtained results has also been proven by means of comparison with real processes.

Section 1.4 presents numerous results of solutions to a broad class of elasticity and thermoelasticity problems within the range of static, quasistatic and dynamic problems. There is also an analysis of the influence of the temperature and deformations' coupling's effect using some examples of thermal and mechanical impacts.

Finally, section 1.5 contains formulation of the equations of coupled dynamic three-dimensional problems with physical non-linearities. Moreover, the finite difference methods, Runge-Kutta's method and the method of additional loads have been combined to form a numerical algorithm of solutions. Convergence of an approximate solution to the real one (the one searched for) has been analysed. The results of problems concerning thermal and mechanical impacts beyond the elasticity fields have been presented and the effects of the influence of reciprocal temperature and deformation fields' coupling on the analysed processes have also been investigated in this chapter.

1.1 Introduction

While designing and constructing electronic devices, industrial facilities, flying objects or technological instrumentation, the problems related to heat processes are particularly important. They appear due to the use of new materials, more complex loads affecting every single element of analysed objects, and also due to an increase of permissible heat loads in devices of smaller and smaller dimensions. As it is generally known, heat processes determine stability of functioning and durability of analysed objects. On the other hand though, numerous empirical measurements of heat processes are extremely complex and expensive. Therefore, exact computational analyses (numerical, as well as analytical) ought to be conducted in order to obtain constructions of optimum characteristics.

In fact, non-stationary temperature reactions in surrounding environment require more accurate calculations than classic modelling of thermomechanical phenomena. In 1845, Duhamel [188] was the first to formulate the theory of elasticity regarding thermal stresses. However it was not until 1956, that Biot [107] introduced a dissipation function into a thermal conduction equation to account for the heat caused by the material's deformation. Thus, the problem of thermoelasticity and the variational principle of coupled theory of thermoplasticity were first formulated. Since then there has been a great interest in that sort of problems.

Earlier works on the theory of thermoelasticity [188] presented a dominating view that a change of temperature within a time interval is small, and therefore it was possible to apply a simplified (quasistatic) method, that is to neglect inertial terms in equations of motion, without the risk of major errors. The next step, introduced by means of the theory of thermoelasticity to simplify the problem, was neglecting dilatation terms in heat conduction equations. Sometimes, when both of the above mentioned terms are neglected in differential equations [598], the solution of a static problem is found. It turns out though, that due to the significance of the problems such simplifications ought not to be made. Among such problems are: the problem of investigating stress waves in deformable bodies; the problems related to determining thermoelastic vibrations; the problems related to investigating stability of conservative elastic systems [119, 164, 267, 316, 356, 466]. In their works, Danilovskoya [160, 161, 162, 163, 164], Kartashova and Shefter [316] analysed the influence of inertial terms on bodies' behaviour considering the inertia forces. They also proved that neglecting a dilatation term does not ensure qualitatively satisfactory results due to inefficient examination of the coupling coefficient's influence on the phenomenon.

All the factors mentioned above caused a growth of interest in complete (i.e. not simplified) problems which fruited in numerous analytical works.

Works of Karlsoy and Eger [315], Lykov [451], Kovalenko [355] and Nowacki [512] contain analyses and generalisation of two, so far independent disciplines, i.e. the theory of elasticity and the theory of heat conduction, and also a definition of so called coupled problem. A full formulation of the principles of variational theories of thermoelasticity is to be found in works [107, 265]. Betti's theorem on reciprocity of virtual works is discussed in monograph [516], and a generalisation of Maizel's method may be found in work [453]. Formulation of flat and space problems of coupled quasistatic theory of thermoelasticity is described in the works of Podstrigach, Schvetz, and Nowacki [512, 516, 545, 546, 547, 548]. Nowacki's monograph [513] introduces equations of the coupled theory of thermoelasticity into wave equations and a method of solving linear and non-linear variants of the problems listed above. Many popular methods of solving the equations of Galerkin's [215] or Papkovich's [528] classic theories of elasticity are generalized in Podstrigach's or Nowacki's works and applied into the theory of coupled thermoelasticity. The method of solving problems of the coupled theory of thermoelasticity in case of a boundless space was proposed by Zorski [727], who used Green's function to solve a heat conduction equation and considered dilatation to be a heat source. Chadwick's work [145] takes up generalized problems of solving boundary problems of the coupled theory of thermoelasticity with the use of integral methods, whereas Souler and Brul use the small parameter method [632].

The problems related to accuracy of formulated boundary problems of the coupled theory of thermoelasticity were described first in book [119], which investigates an initial boundary problem for an isotropic body, later extended also onto an anisotropic body in Ionescu work [277].

Numerous dynamical problems of mathematical physics apply various integral transformations, including Laplace's transformation [294], the solution of which is related to the use of Fourier's series. In their work, Kupradze and others [398] propose their theory of multidimensional singular integral equations that makes it possible to investigate the static and dynamic problems of stabilised continuous systems' vibrations. Hybrid problems, investigated by Magnaradze [452], Kupradze and Burchuadze [397] may be solved with generalized integrals that correspond to differential equations with the use of harmonic and analytical functions.

Defermos' work [175] contains many theorems concerning basic problems of the theory of thermoelasticity, including their proofs. Work [101] investigates the so-called second and third boundary and initial boundary problems of the coupled theory of thermoelasticity with the use of the method of potential and Laplace's transformation. Work [397] analyses four basic three-dimensional boundary problems of the theory of thermoelasticity in case of harmonic vibrations of a homogeneous isotropic medium with the following conditions set in its boundaries: