

VLADIMIR I. ARNOLD

# Collected Works

VOLUME II

Hydrodynamics, Bifurcation Theory,  
and Algebraic Geometry  
1965–1972

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Vladimir I. Arnold, drawing, 1968.

*Photo by Dmitry Arnold*

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### VOLUME II

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Edited by

Alexander B. Givental  
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Vladimir I. Arnold  
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# Preface

This volume of the Collected Works appears in print after Vladimir Arnold's untimely death in June 2010. His passing was a terrible loss for mathematics and science in general.

We hope that this project of Collected Works, which is needed now more than ever, will contribute to establishing the tremendous legacy of V.I. Arnold, a remarkable mathematician and human being. Some memories of V.I. Arnold can be found in the recent March and April 2012 issues of the Notices of the AMS.

Our Editorial team has also suffered an unprecedented blow since Volume I was published in 2009. Jerry Marsden passed away in September 2010, and Vladimir Zakalyukin passed away in December 2011. We dedicate this volume to their memory.

This Volume II of the Collected Works includes papers written by V.I. Arnold mostly during the period from 1965 to 1972. This was an amazingly productive period, starting with a year Arnold spent in Paris. During this period he made fundamental contributions to the fields of hydrodynamics, algebraic geometry, singularity and bifurcation theories, and dynamical systems. We have also added later papers by Arnold on related topics so as to make this volume more complete and comprehensive.

Several papers were translated from Russian specifically for this volume. Unfortunately it was not possible to replace certain previously existing translations, completed in the era of the Iron Curtain, when it could not be expected that translators understood the subject. As an alarming example, we refer the reader to the paper “Local problems of analysis”, and to the editors' comment therein. As a counter-example, see the paper “On the arrangements of ovals...”, which has been translated once again.

For the same reason some of the titles in the reprinted translations were incorrect. Since it was not possible to correct them in the reprinted articles, we have given their correct versions in the Contents along with the versions used in the reprinted contributions.

November 2012

Alexander Givental  
Boris Khesin  
Alexander Varchenko  
Victor Vassiliev  
Oleg Viro

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1. (with A. Avez): “Problèmes ergodiques de la mécanique classique” (in French), Monographies Internationales de Mathématiques Modernes, No. 9, Gauthier-Villars, Paris, 1967, ii+243pp.; English translation: “Ergodic problems of classical mechanics”, W. A. Benjamin, Inc., New York-Amsterdam, 1968, ix+286pp.
2. “Obyknoennyye differentsialnye uravneniya” (in Russian) Nauka, Moscow, 1971, 239pp.; English translation: “Ordinary differential equations”, The M.I.T. Press, Cambridge, 1973, ix+280pp.; Other translations/editions of this book are French (1984), Russian (2<sup>nd</sup> edition, unrevised, 1975), Russian (3rd edition, revised and extended, 1984), Polish (1975, 1983), Portuguese (1985), German (1980), etc.
3. Editor’s preface to a collection of papers “Singularities of differentiable mappings”, Mir, Moscow, 1968.
4. Translator’s preface to F. Pham’s book “Introduction to the topological study of Landau’s singularities”, Mir, Moscow, 1970.
5. Translator’s preface to J. Milnor’s “Singular points of complex hypersurfaces”, Mir, Moscow, 1971.